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Trade Effects on the Personal Distribution of Wealth

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M E X I C O

Trade Effects on the Personal Distribution of Wealth *

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ABSTRACT

This paper develops a dynamic Heckscher-Ohlin model and studies the interaction between international trade and wealth distribution dynamics. I also study how differences in the cost of financial intermediation among countries may affect the pattern of trade and wealth dynamics. Relative to the inequality it would have prevailed under autarky, I find that trade promotes a decline (an increase) in inequality when the economy converges to the steady state form below (above). However, with trade inequality increases (declines) during the transition from below (above). I also find that trade may alleviate frictions in the financial intermediation sector in economies where these frictions are larger. In those economies, trade may in fact promote a higher income than under autarky.

KEYWORDS: International Trade, Wealth Distribution, Financial Intermediation
JEL CLASSIFICATION: E21, F11, F17

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1 Introduction

In the theoretical literature on international trade, the Stolper-Samuelson theorem stands as one of the main results about income distribution. Roughly speaking, the theorem states that with the opening to international trade, prices of relatively abundant factors increase and prices of relatively scarce factors decrease. This change in factor prices is the consequence of a more specialized production in goods that use intensively relatively abundant factors.[1] Thus, the theorem describes the changes in the *functional distribution* of income of an economy that opens to international trade.[2] Changes in the functional distribution of income would translate one-to-one in changes in the *personal distribution* of income in some economies, for instance, in economies inhabited by "pure workers" and "pure capitalists". However, it is not obvious how these changes may affect personal inequality in income and/or wealth in less polarized societies. Perhaps surprisingly, the effects of international trade on the personal distribution of income have received little attention in the theoretical literature.[3] In stead, how development and growth may affect income and wealth inequality is an old theme in macroeconomics (see, for instance, Kuznets (1955), and Stiglitz (1969)). Furthermore, recently there is a large literature showing that intermediation costs and other frictions in the financial sector have sizable effects on growth rates and steady states.[4] What are the international trade effects on income/wealth inequality among households? How does international trade in goods affect the role of the financial intermediation sector? How do frictions in financial intermediation interact with trade and wealth dynamics? The goal of this paper is to provide an answer to these questions.

In this paper I extend the study by Chatterjee (1994) about the dynamics of the distribution of income and wealth in a standard one-sector neoclassical model of growth to the case of a dynamic Heckscher-Ohlin model of international trade. Chatterjee

[1] There are several empirical applications of the Stolper-Samuelson theorem as an explanation for observed wage differentials between skilled and unskilled workers in trading economies. The empirical evidence supporting the theorem is mixed. For example, Wood (1997) reports that the skill premium narrowed in some East Asian countries during the 1960s and 1970s, while it widened in Latin American countries during the 1980s and 1990s. See also the results in Robertson (2001) for the specific case of Mexico after the NAFTA.

[2] Ripoll (2000) develops a three-good, three-factor, dynamic model and shows that in addition to relative abundance of factors, the timing of the opening to trade has sizable effects on steady states and on the dynamic path of factor prices.

[3] This is not so in the empirical literature, to which I will briefly refer below. Among theoretical studies, Fischer and Serra (1996), and the more closely related paper by Das (2000). The main differences between the study by Das and mine are that he develops an overlapping generations model with imperfect competition in production, whereas I assume infinitely lived agents and perfect competition in all markets. Also, in his model capital is tradeable, whereas in my model it is not.

[4] Levine (1997) contains many references and he also reports some evidence about the differences in the financial sector between rich and poor countries; see also the more recent survey Smith (2002).

(1994) showed that in an economy where agents differ only in their initial wealth, the transition towards the steady state from below (from above) may have a negative (positive) effect on the degree of lifetime wealth/income inequality prevailing in the economy.[5] His findings are relevant in the study of the effects of international trade because in a dynamic model, trade is likely to give rise to a different steady state than under autarky. Following Atkeson and Kehoe (2000), I analyze the case of a small open-economy that trades goods with the rest of the world at equilibrium prices in international markets. In particular, I assume that all economies are identical and that the only difference among them is that the rest of the world is already at the steady state. Because of the small open-economy assumption, trade has no effects on the distribution of wealth in the rest of the world. Nevertheless, the distribution of wealth in the small economy does change over the transition to the steady state. Over a transition to the steady state from below (when the initial stock of capital is smaller than its long run level), these changes in the small open-economy are the result of two conflicting effects: an "international trade" effect, which tends to reduce inequality in the functional distribution of income, and a "transition" effect that tends to increase inequality in the personal distribution of income and wealth (the opposite holds over a transition from above). I show that if the opening to trade occurs once the small economy is sufficiently close to its steady state under autarky from below (from above), personal inequality in wealth falls to a permanently lower level (jumps to a permanently higher level) than under autarky. Then I use numerical methods to study the long run level of inequality when the economy opens to international trade far away from the steady state (i.e., starting from arbitrary levels of capital). I find that inequality in wealth and income increases (declines) over the transition from below (above) to the steady state with trade. However, inequality is always smaller (larger) than under autarky. In this sense, my results suggest that the "international trade" effect dominates the "transition effect". I also look at timing effects of the opening to international trade. My results suggest that, irrespectively of whether the economy converges to the steady state from above or from below, the sooner an economy opens to international trade, the smaller will be the level of inequality in the long run.

I extend the previous results and investigate how international trade affects the role of the financial intermediation sector, and how differences in this sector among countries may affect the pattern of trade and the evolution of inequality.[6] Differences in the financial intermediation sector are introduced in the model by different depreciation

[5]Caselli and Ventura (2000) extend these results in a continuous time model with additional sources of heterogeneity (preferences and labor productivity); see also Sorger (2000) where the effect of a leisure/labor decision is studied.

[6]Acemoglu and Ventura (2001) also consider differences in the financial intermediation sector in their study of the world income distribution.

rates of capital and different transaction costs in the rest of the world and in the small open-economy. I show that small economies where costs of financial intermediation are large, tend to completely specialize production in labor intensive goods. Thus, the differences in the financial intermediation sector have sizable effects on the determination of the pattern of production and international trade. This result holds in the long run even if the small economy has initially comparative advantage in the production of capital intensive goods, i.e., its capital labor ratio is larger than in the rest of the world. Thus, an interesting implication of the differences in intermediation costs is that the opening to trade may entail a decline in the stock of capital. This means that a small economy that has been growing for some time, may start depleting capital after the opening to international trade. Over such a transition, wealth inequality would increase at first, suddenly jump at the moment of opening the economy, and then continue declining towards the steady state level prevailing under trade. Therefore, differences in the cost of financial intermediation and trade may reverse the pattern of capital accumulation and wealth dynamics.

It is well known that the steady state level of capital of an economy completely specialized in the production of labor-intensive goods is smaller than it would under autarky (see Atkeson and Kehoe (2000)). Interestingly, I show that the real income at the steady state with trade may be larger than under autarky if the financial intermediation cost in the economy is sufficiently large compared to that in the rest of the world. In other words, international trade alleviates the effects of costly financial intermediation in economies where these costs are larger. This finding is interesting because the combination of differences in the cost of financial intermediation with trade may provide a smaller amount of capital than under autarky, a larger income than under autarky, and over the transition, wealth inequality would be declining. Finally, I show that international trade may "immiserize" an economy where transaction costs in the financial sector are only slightly larger than in the rest of the world. This result holds even if there are no "terms of trade" effects, as prices of goods are determined in the rest of the world and they are constant.

The results I just described suggest that in general, the opening to international trade does not necessarily imply a decline, or a rise, in inequality of the personal distribution of wealth. In this sense, my results are consistent with the available empirical evidence. For instance, Wei and Wu (2001) find that openness to trade and urban-rural inequality are negatively associated in Chinese cities; Edwards (1997) reports that trade reforms do not seem to affect income distribution, and Litwin (1998) finds that trade openness in general worsen income distribution.[7] Also, Sala-i-Martin (2002) finds that income

[7] On the related issue about growth an inequality, Edwards (1997) also reports some evidence suggesting that countries that grew faster observed an increase in inequality. Deininger and Squire (1996, pg. 588) find "no systematic relationship between growth of aggregate income and changes

inequality within-countries has increased over the period of globalization, although this effect is too small to offset the large decrease in across-country disparities. At the end of the paper I briefly discuss an extension of the current model that could help to empirically assess more sharply the effects of growth, and possibly trade, on wealth inequality.

The paper continues as follows: Section 2 introduces a world with many competitive economies, section 3 describes equilibrium dynamics under autarky and shows that the results in Chatterjee (1994) can be extended to two-sector economies. Section 4 studies the effects of trade on the distribution of wealth of a small open-economy. Finally section 5 investigates the effects of differences in the financial intermediation sector on the pattern of trade and wealth dynamics, and section 6 concludes. An appendix at the end of the paper contains proofs and a description of the numerical methods used in section 3.

2 The model

There is a large number of small economies. These economies are identical in all respects except perhaps in the initial distribution of capital and in the technology available to transfer resources across time. A typical economy is described in the next sections. In what follows capital characters denote aggregate variables, lower case characters denote per capita variables and individual variables are denoted by lower case characters with a script i.

2.1 Production

In each economy production is organized in two sectors, one producing a final good that can be devoted to consumption and investment, and the other producing intermediate goods used as inputs in the final goods sector. In the intermediate goods sector there are two industries producing goods x and y. In each industry there is a large number of perfectly competitive firms that take goods and factor prices as given. In this sector technologies for production display constant returns to scale and the only difference between them is that they use primary inputs in different intensities:

$$X_t = K_{x,t}^{\theta} L_{x,t}^{1-\theta},\ Y_t = K_{y,t}^{\eta} L_{y,t}^{1-\eta},\ \text{ with } \theta, \eta \in (0,1). \tag{1}$$

In the previous equations the subindex x and y on the primary factors indicate amounts used in the production of each good. Assuming $\theta > \eta$ then the production of x is capital

in inequality as measured by the Gini coefficient", meaning that along the growth path, inequality increases in some countries while it declines in some others.

intensive. Furthermore, by assuming Cobb-Douglas technologies I am also ruling out factor intensity reversals, thus for all possible relative factor prices the capital labor ratio in industry x will be larger than that in industry y. Firms in these industries rent capital and labor in each period so as to maximize profits, thus the optimal decisions are characterized by the following conditions:

$$r_{x,t} \geq p_{x,t}\theta K_{x,t}^{\theta-1} L_{x,t}^{1-\theta}, r_{y,t} \geq p_{y,t}\eta K_{y,t}^{\eta-1} L_{y,t}^{1-\eta}, \tag{2}$$

with equality whenever $K_{j,t} > 0$ for $j = x, y$ and

$$w_{x,t} \geq p_{x,t}(1-\theta) K_{x,t}^{\theta} L_{x,t}^{\theta-1}, w_{y,t} \geq p_{y,t}(1-\eta) K_{y,t}^{\eta} L_{y,t}^{\eta-1}, \tag{3}$$

with equality whenever $L_{j,t} > 0$ for $j = x, y$ and where $r_{j,t}$ and $w_{j,t}$ stand respectively for the rental rate of capital and labor in each industry in period t.[8]

The final goods sector is also perfectly competitive. The technology displays constant returns to scale and uses as inputs intermediate goods only: $Z_t = X_t^{\gamma} Y_t^{1-\gamma}$, with $\gamma \in (0,1)$. Normalizing the price of the consumption/investment good to one, optimal decisions for profit maximization of firms in this sector are characterized by the following first order necessary conditions:

$$p_{x,t} = \gamma X_t^{\gamma-1} Y_t^{1-\gamma}, \text{ and } p_{y,t} = (1-\gamma) X_t^{\gamma} Y_t^{-\gamma}. \tag{4}$$

2.2 People

Each economy is inhabited by N agents indexed by $i = 1, 2, 3, \ldots N$. Each of these agents behaves so as to maximize the present value of the utility derived from the consumption of the homogeneous final good over an infinite horizon:

$$\sum_{t=0}^{\infty} \beta^t u(c_t^i), \tag{5}$$

where $\beta \in (0,1)$ is the subjective discount factor, the same for all agents. In the rest of the paper it will be assumed that preferences take the form of $u(c_t^i) = \log(\alpha + c_t^i)$, where α is a real number (the same for all agents). In case $\alpha < 0$ then marginal utility of consumption can be arbitrarily large even for strictly positive levels of consumption. The interpretation in this case is that there is a minimum consumption level and it will be required that $\alpha + c_t^i \geq 0$. As shown in Chatterjee (1994), the implications of

[8] In the previous expressions I take into account that complete specialization is possible with international trade, thus equilibrium prices may not coincide with the value of marginal productivity of factors in all sectors.

trade on the personal distribution of wealth I derive in this paper will also hold under a more general class of utility functions.[9]

2.3 Endowments

Agents are endowed with k_0^i units of productive capital in the first period. The initial endowment of capital is the only difference among agents. To transfer capital across periods agents have access to the following investment technology:

$$k_{t+1}^i = i_t^i + (1-\delta)k_t^i. \tag{6}$$

In the previous equation $\delta \in (0,1)$ is the depreciation rate of capital. In addition to the initial endowment of capital, in the beginning of each period agents receive a perfectly divisible unit of time which they inelastically supply as labor. Both capital and labor are freely mobile across firms in the intermediate goods sector.

2.4 An agent's problem

For the maximization of the objective in Equation (5), agents collect capital and labor incomes and decide current consumption and how much to invest or desinvest to obtain the desired stock of capital for the next period. The following budget constraint formalizes the set of possible consumptions and investments for an agent starting a period t with capital k_t^i:

$$c_t^i + i_t^i = \sum_j (w_{j,t} l_{j,t}^i + r_{j,t} \nu_{j,t}^i k_t^i), \;\; j = x, y. \tag{7}$$

In the previous equation $l_{j,t}^i$ and $\nu_{j,t}^i$ stand respectively for the fraction of labor and capital supplied to the j industry, thus $l_{x,t}^i + l_{y,t}^i = 1$ and $\nu_{x,t}^i + \nu_{y,t}^i = 1$. The budget constraint can be simplified by noticing that if both intermediate goods industries are in operation, then perfect competition in factor markets and free mobility of primary factors ensures that equilibrium rental rates of capital and labor will be the same in both industries. If, alternatively, only one industry is active, then it will also be the case that there will be a unique market wage and interest rate of equilibrium. Thus given equilibrium factor prices agents are indifferent with respect to where to supply capital and labor. Taking these facts into account the problem agents need to solve

[9] This class includes $u(c) = \rho(\alpha + \psi c)^\sigma$ with a) $\sigma < 1$ but different from zero, $\rho = 1/\sigma$, $\psi = 1$ and α a real number, or b) $\sigma = 2$, $\rho = -1/2$, $\psi = -1$ and $\alpha > 0$. It also includes the case of $u(c) = -\alpha \exp(-\psi c)$ with both α and ψ strictly positive.

can be written formally as follows:

$$\begin{array}{ll} \max & \sum_{t=0}^{\infty} \beta^t \log(\alpha + c_t^i) \\ \text{s. to} & c_t^i + i_t^i = w_t + r_t k_t^i, \\ & k_{t+1}^i = i_t^i + (1-\delta) k_t^i, \\ & c_t^i \geq \max\{0, -\alpha\},\ k_t^i \geq 0,\ \forall t \geq 0,\ \text{given } k_0^i. \end{array} \tag{8}$$

In the previous problem agents take prices as given and it is assumed that the initial condition for capital is large enough so that the solution for consumption is interior. When this is the case the first-order necessary condition for optimality is given by

$$\frac{1}{\alpha + c_t^i} = \beta R_{t+1} \frac{1}{\alpha + c_{t+1}^i}, \tag{9}$$

where $R_{t+1} = r_{t+1} + 1 - \delta$. Let lifetime wealth of agent i in a period t be given by

$$\omega_t^i = R_t \left[k_t^i + \sum_{j=0}^{\infty} \frac{w_{t+j}}{\prod_{s=0}^{j} R_{t+s}} \right]. \tag{10}$$

Notice that ω_t^i is composed of the real value of capital at the end of the period plus the real present value of labor. This measure of wealth is useful for the purposes of this paper because it captures changes in factor-prices, and variations in goods and factor prices are a central issue in international trade theory. Repeated substitutions of Equation (9) in the budget constraint in (8) and the use of the previous definition provides the following expression for an agent's consumption in period t:

$$c_t^i = (1-\beta)\omega_t^i + a({}_tR), \tag{11}$$

where $a({}_tR) = -\alpha \sum_{j=0}^{\infty} \frac{\beta R_{t+1+j} - 1}{\prod_{s=0}^{j} R_{t+1+s}}$.

In the following section I describe the equilibrium dynamics under autarky for one of the economies.

3 Competitive equilibrium under autarky

For any of the previous economies a competitive equilibrium is a list of sequences $\{p_t^x, p_t^y, w_t, r_t\}$ such that markets for primary factors and intermediate goods clear and that the aggregation of optimal decisions of agents satisfy the following market clearing condition for final goods:

$$\frac{\sum_N c_t^i + k_{t+1}^i}{N} = x_t^{\gamma} y_t^{1-\gamma} + (1-\delta) \frac{\sum_N k_t^i}{N}, \tag{12}$$

where x_t and y_t stand for the production of intermediate goods in per capita terms. Notice that since both intermediate goods are fundamental in the production of the final good, in equilibrium they will be produced at any given period. It follows that

$$\frac{p_{x,t}}{p_{y,t}} = \frac{\gamma}{1-\gamma}\frac{y_t}{x_t}. \tag{13}$$

Let l_t be the fraction of total labor used in the production of x in a period t. Let also ν_t be the fraction of per capita capital used in the production of x. Then the stock of capital in per capita terms used in the production of each intermediate good can be written as $k_{x,t} = \nu_t k_t$ and $k_{y,t} = (1-\nu_t)k_t$. Since primary factors are perfectly mobile across industries, in equilibrium it must be the case that $w_{x,t} = w_{y,t}$ and that $r_{x,t} = r_{y,t}$. Using Equation (13) in the equilibrium condition $r_{x,t} = r_{t,y}$, a few manipulations produce

$$\frac{\eta}{\theta}\frac{1-\gamma}{\gamma} = \frac{1-\nu_t}{\nu_t}. \tag{14}$$

It follows that the fraction of capital devoted to the production of x is constant over time and is given by $\nu^* = \theta\gamma/(\eta(1-\gamma)+\theta\gamma)$. Using the fact that prices of primary factors are the same in both industries and dividing r_t by w_t in (2) and (3) produces

$$\frac{\theta}{1-\theta}\frac{l_t^x}{\nu^*} = \frac{\eta}{1-\eta}\frac{1-l_t^x}{1-\nu^*}. \tag{15}$$

Therefore the fraction of labor devoted to the production of the intermediate good x is also constant over time and is given by $l^* = (1-\theta)\gamma/((1-\eta)(1-\gamma)+(1-\theta)\gamma)$.[10] With the expressions for l^* and ν^* the market clearing condition for final goods can be written as

$$\frac{\sum_N c_t^i + k_{t+1}^i}{N} = Ak_t^{\xi} + (1-\delta)\frac{\sum_N k_t^i}{N}, \tag{16}$$

where A and ξ are constants involving l^* and ν^* and other parameters in the production technologies.[11] Finally, since individual consumption in Equation (11) is linear in wealth, then consumption per capita is independent of the distribution of wealth. Since wealth in Equation (10) is linear in capital, it follows from the previous equation that per capita capital depends only on the consumption of an agent that has the average capital in the economy. Therefore the evolution of capital over time can be studied by means of the problem of a central authority that solves

$$\begin{array}{ll} \max & \sum_{t=0}^{\infty}\beta^t \log(\alpha + c_t) \\ \text{s. to} & c_t + k_{t+1} = Ak_t^{\xi} + (1-\delta)k_t \\ & c_t \geq \max\{0,-\alpha\},\ k_t \geq 0,\ \forall t \geq 0,\ \text{given } k_0. \end{array} \tag{17}$$

[10]Notice therefore that in the autarky equilibrium where the economy produces both intermediate goods the equilibrium ν_t and l_t are always given by ν^* and l^*, independently of the available amount of capital. Also, $\theta > \eta$ implies that $l^*/\nu^* < 1$. This fact will be relevant in section 5 below.

[11]Specifically, $A = (\nu^*/l^*)^{\theta\gamma}((1-\nu^*)/(1-l^*))^{\eta(1-\gamma)}l^{*\gamma}(1-l^*)^{1-\gamma}$ and $\xi = \theta\gamma + \eta(1-\gamma)$.

This problem is a version of the neoclassical model of growth studied at length in the literature. Since α can be negative I will assume that the initial stock of capital is larger than some lower bound $\underline{k} \geq 0$ so that the feasible set is not empty. I will also assume that $\underline{k} < k^*$.[12] The following proposition states some well known properties of the solution to the previous problem and thus is stated without proof.

Proposition 1. Under the maintained assumptions if $k_0 > \underline{k}$ then there exists a sequence $\{c_t, k_{t+1}\}$ that solves the planner's problem. The sequence $\{c_t, k_{t+1}\}$ monotonically converges to stationary values $\{c^*, k^*\}$ satisfying (i) $A\xi(k^*)^{\xi-1} - \delta = (1-\beta)/\beta$, and (ii) $c^* = A(k^*)^{\xi} - \delta k^*$.

The solution for capital from the previous proposition can be used to recover all equilibrium prices using Equations (2), (3) and (13), and with them, it is possible to study the dynamics of the personal distribution of wealth.[13] From the budget constraint of an agent and the definition of wealth it is easy to see that total wealth of an agent evolves according to

$$\frac{\omega_{t+1}^i}{\omega_t^i} = R_{t+1}\left(1 - \frac{c_t^i}{\omega_t^i}\right). \tag{18}$$

Therefore the evolution of personal wealth depends on the consumption to wealth ratio. Also, using Equation (11) one obtains that

$$\frac{c_t^i}{\omega_t^i} < (\geq) \frac{c_t}{\omega_t} \Leftrightarrow a({}_tR)(\omega_t^i - \omega_t) > (\leq) 0.$$

The previous observation and Equation (18) imply that if an agent's wealth is larger than per capita wealth and the term $a({}_tR)$ is positive, then that agent will accumulate wealth at a faster rate than the rest of the economy, i.e., his wealth share will increase over time. The measure of inequality in wealth I will use is Lorenz-dominance on wealth shares (denoted s_t^i), which I define next.

Definition. Order agents according to increasing wealth. A vector $\{s_t^i\}$ is said to Lorenz-dominate a vector $\{s_{t+1}^i\}$ if $\sum_i^K s_{t+1}^i \leq \sum_i^K s_t^i$ for all $1 \leq K \leq N$, with strict inequality for some K.

With this notation in place the dynamics of the distribution of wealth are completely described by the following proposition:

[12] If $\alpha \geq 0$, then $\underline{k} = 0$. For $\alpha < 0$ the feasible set will be empty unless there exists a solution to $Ak^{\xi} + \alpha - \delta k = 0$. I will assume that the previous equation has two solutions an that the smaller one satisfies $\underline{k} < k^*$, where k^* is the unique k satisfying the following condition *(i)* in the text.

[13] The rest of this section introduces some notation for individual wealth and wealth shares before I state a theorem due to Chatterjee (1994).

Proposition 2 (Chatterjee 1994, pag. 104). $a({}_tR) > (\leq) 0 \Leftrightarrow \alpha(k_t - k^*) > (\leq) 0$. *Thus if* $\alpha(k_t - k^*) > (<)\, 0$, *then* $\{s_t^i\}$ *Lorenz-dominates (is Lorenz-dominated by)* $\{s_{t+1}^i\}$. *Furthermore, if* $\alpha(k_t - k^*) = 0$, *then* $\{s_t^i\} = \{s_{t+1}^i\}$.

I will refer to a situation where $k_0 < k^*$ and thus, the stock of capital will be growing over time, as a *transition from below*. Conversely, a situation where the stock of capital decreases over time towards its steady state level will be called a *transition from above*. Thus, an implication of Proposition 2 is that over a transition from below the distribution of personal wealth becomes more unequal over time if there is a subsistence requirement ($\alpha < 0$). In the following section I study the implications of the previous proposition once the economies engage in international trade in intermediate goods.

4 Wealth dynamics in a small open-economy

Following Atkeson and Kehoe (2000), I study the effects of trade on the distribution of lifetime wealth of a small economy that starts its development process once the rest of the world has reached the steady state. In particular I assume that all but one economy started growing towards the steady state at the same time and with the same level of initial capital. This means that whether these economies were allowed to trade in intermediate goods over the transition to the steady state is of no consequence. Therefore the equilibrium dynamics for these economies are described by Propositions 1 and 2 above: all economies converge to the same steady state independently of the initial distribution of wealth, and at the steady state the only difference among them is in the stationary distribution of wealth.

Consider now the equilibrium dynamics of the economy that starts its process of development once the rest of the world has reached the steady state. This economy can trade intermediate goods at the international equilibrium prices. Since aggregate dynamics do not depend on the initial distribution of wealth, the evolution of this economy can be described by the decisions of a central authority that buys and sells intermediate goods in international markets and that organizes in an efficient way domestic production. As in the preceding section, it is useful to write the problem in per capita terms so that the planner chooses the fraction of available capital and labor to be devoted to the production of each intermediate good. Using the notation introduced so far the problem of the central authority can be described as

$$
\begin{array}{ll}
\max & \sum_{t=0}^{\infty} \beta^t \log(\alpha + c_t) \\
\text{s. to} & c_t + k_{t+1} = (x_t^d)^\gamma (y_t^d)^{1-\gamma} + (1-\delta) k_t \\
& x_t^s = (\nu_t k_t)^\theta l_t^{1-\theta}, \\
& y_t^s = ((1-\nu_t) k_t)^\eta (1-l_t)^{1-\eta}, \\
& p_x^* x_t^d + p_y^* y_t^d = p_x^* x_t^s + p_y^* y_t^s, \\
& c_t \geq \max\{0, -\alpha\},\ k_t \geq 0,\ l_t \in [0,1], \nu_t \in [0,1]\, \forall t \geq 0, \\
& \text{given } k_0.
\end{array}
\tag{19}
$$

In the previous problem the first equation is the feasibility constraint that restricts consumption and capital accumulation to the sum of current output and undepreciated capital; the second and third equations simply relate the domestic supply of intermediate goods to the technology constraints; the forth equation is the condition for balance in international trade. After substituting domestic supply of intermediate goods in the balanced trade equation, the following equations describe the first order conditions that the solution of the previous problem must satisfy:

$$
\begin{array}{ll}
x_t^d: & \mu_t \gamma (x_t^d)^{\gamma-1} (y_t^d)^{1-\gamma} - \lambda_t p_x^* \leq 0, x_t^d \geq 0, \\
y_t^d: & \mu_t (1-\gamma)(x_t^d)^\gamma (y_t^d)^{-\gamma} - \lambda_t p_y^* \leq 0, y_t^d \geq 0, \\
\nu_t: & \lambda_t \left[p_x^* \theta \nu_t^{\theta-1} k_t^\theta l_t^{1-\theta} - p_y^* \eta (1-\nu_t)^{\eta-1} k_t^\eta (1-l_t)^{1-\eta} \right] - \lambda_t^\nu \leq 0, \nu_t \geq 0, \\
l_t: & \lambda_t \left[p_x^* (1-\theta) \nu_t^\theta k_t^\theta l_t^{-\theta} - p_y^* (1-\eta)(1-\nu_t)^\eta k_t^\eta (1-l_t)^{-\eta} \right] - \lambda_t^l \leq 0, l_t \geq 0, \\
k_{t+1}: & -\mu_t + \beta \mu_{t+1}(1-\delta) + \beta \lambda_{t+1} \left[p_x^* \theta \nu_{t+1}^\theta k_{t+1}^{\theta-1} l_{t+1}^{1-\theta} \right. \\
& \left. + p_y^* \eta (1-\nu_{t+1})^\eta k_{t+1}^{\eta-1} (1-l_{t+1})^{1-\eta} \right] \leq 0, k_{t+1} \geq 0,
\end{array}
\tag{20}
$$

where $\mu_t, \lambda_t, \lambda_t^\nu$ and λ_t^l stand for the non negative Lagrange multipliers associated to the feasibility constraint, balance in trade and feasibility for ν_t and l_t respectively. A version of Proposition 1 can be shown to apply to the current planner's problem. In particular, Atkeson and Kehoe (2000) show that the steady state of the late-bloomer depends on the initial endowment of capital. To see this in the current discrete time version of the model, assume for a moment that for all k the solutions for l and ν are interior and thus the corresponding first order conditions are satisfied with equality. It follows that

$$
\frac{\nu_t}{l_t} = \frac{\theta(1-\eta)}{(1-\theta)\eta(1-l_t) + \theta(1-\eta) l_t}. \tag{21}
$$

Inserting the previous equation in the first order condition for ν produces

$$\left[\frac{p_x^*}{p_y^*}\left(\frac{\theta}{\eta}\right)^{\eta}\left(\frac{1-\theta}{1-\eta}\right)^{1-\eta}\right]^{1/(\eta-\theta)} = \frac{k_t\theta(1-\eta)}{(1-\theta)\eta(1-l_t)+\theta(1-\eta)l_t}. \tag{22}$$

The implication of the previous equation is that for given intermediate goods prices of equilibrium in international markets, if the initial condition for capital is sufficiently small then the economy specializes in the production of the labor intensive good. Conversely, if the economy starts out with a sufficiently large amount of capital, then it will specialize in the production of the capital intensive good. Therefore there is an interval $[k_y, k_x]$ known as the *cone of diversification* such that if $k_0 \notin [k_y, k_x]$ the late-bloomer specializes production completely.[14] It is straightforward to show that the interval $[k_y, k_x]$ constitutes the set of rest points for the late-bloomer.[15] Therefore if $k_0 < k_y$ then $\beta R_0 > 1$ and the economy keeps accumulating capital over time and converges to k_y. Over this transition Proposition 2 applies and after the opening to trade the distribution of wealth becomes more unequal if $\alpha < 0$ and more equal if $\alpha > 0$. Similarly, if $k_0 > k_x$ then $\beta R_0 < 1$, thus after the opening to trade the stock of capital decreases over time converging to k_x and over the transition inequality decreases (increases) if $\alpha < 0$ ($\alpha > 0$). If the initial stock of capital lies in the interval $[k_y, k_x]$, then there is a reallocation of primary factors but the economy does not initiate a transition to a new steady state. For $k_0 \in [k_y, k_x]$ Proposition 3 below describes the changes in the personal distribution of wealth after the opening to trade assuming $\alpha < 0$. The proof of Proposition 3 uses Lemma 1.

Lemma 1. Let $\hat{k}_1 = k^*((1-\nu^*)/(1-l^*))^{(1-\eta)/(\gamma(\theta-\eta))}$ *and* $\hat{k}_2 = k^*(\nu^*/l^*)^{\theta/((\theta-\eta)(1-\gamma))}$. *Then* $\hat{k}_1 < k_y$ *and* $k_x < \hat{k}_2$, *and if* $\hat{k}_1 \leq k_t < k^*$ ($k^* < k_t \leq \hat{k}_2$) *then* $w_t \leq \hat{w}_t$ *and* $r_t \geq \hat{r}_t$ ($w_t \geq \hat{w}_t$ *and* $r_t \leq \hat{r}_t$).

Proof: See the Appendix.

In the previous lemma w_t, r_t stand respectively for the equilibrium wage rate and interest rate corresponding to a given stock of capital in operation in a period t if the economy remains under autarky, and $\hat{w}_t$ and $\hat{r}_t$ are the corresponding factor-

[14] The end points in the interval are given by $k_y = (1-\theta)\eta/(\theta(1-\eta))((p_x^*/p_y^*)(\theta/\eta)^{\theta}((1-\theta)/(1-\eta))^{1-\theta})^{1/(\eta-\theta)}$ and $k_x = ((p_x^*/p_y^*)(\theta/\eta)^{\eta}((1-\theta)/(1-\eta))^{1-\theta})^{1/(\eta-\theta)}$.

[15] To see this, notice that when the late-bloomer specializes completely in the production of y the corresponding first order condition in Equation (2) implies that $r = p_y^*\eta k_y^{\eta-1}$. For the early-bloomers the corresponding expression is given by $r = p_y^*\eta(k^*(1-\nu^*)/(1-l^*))^{\eta-1}$. Since early bloomers produce both intermediate goods, it follows from Equation (4) at the steady state that $p_x^*/p_y^* = \gamma/(1-\gamma)Bk^{*\eta-\theta}$, where $B = ((1-\nu^*)/(1-l^*))^{\eta}(l^*/\nu^*)^{\theta}(1-l^*)/l^*$. Using the definitions for l^*, ν^* and k_y given above, it follows that $k_y = k^*(1-\nu^*)/(1-l^*)$. Therefore k_y is in fact a rest point for the late-bloomer. A similar argument applies to show that $k_x = k^*\nu^*/l^*$, thus k_x is also a rest point, as well as convex combinations of k_y and k_x.

prices if the economy opens to trade in that period. Therefore Lemma 1 states that a version of the Stolper-Samuelson theorem holds for $k_0 \in [\hat{k}_1, \hat{k}_2]$. If the economy is far away from the steady state under autarky then $r_t < \hat{r}_t$ when $k_t < \hat{k}_1$ ($r_t > \hat{r}_t$ if $k_t > \hat{k}_2$), and yet $\hat{w}_t/\hat{r}_t > w_t/r_t$ when the economy specializes in the labor intensive good ($\hat{w}_t/\hat{r}_t < w_t/r_t$ when it specializes in the capital intensive good). Thus the Stolper-Samuelson theorem still holds, in the sense that the income accruing to the relatively abundant factor increases more than the income accruing to the relatively scarce factor.[16]

Proposition 3. Assume $\alpha < 0$, order agents according to increasing wealth and let $\bar{i}$ be the "name" of the agent with the largest wealth share smaller or equal to $1/N$. If $k_y \leq k_0 < k^*$ ($k^* < k_0 \leq k_x$), then
3.1: $\hat{s}_0^i \geq (\leq) s_0^i$ for $i \leq \bar{i}$ and $\hat{s}_0^i < (>) s_0^i$ for $i > \bar{i}$,
3.2: $\{\hat{s}_t^i\} = \{\hat{s}^i\}$ $\forall t \geq 0$,
3.3: $\{\hat{s}^i\}$ Lorenz-dominates (is Lorenz-dominated by) $\{s_t^i\}$ $\forall t \geq 0$.

Proof: See the Appendix.

The intuition behind 3.1 is that when the stock of capital is sufficiently close to its steady state level under autarky, the change in factor prices following the opening to international trade benefits (worsen) more those agents for whom labor income represents a larger fraction of their wealth if the economy is approaching the steady state from below (from above). Specifically, the opening to trade "compresses" (expands) the distribution of wealth shares but preserves its ordering. 3.2 says that inside the cone of diversification the personal distribution of wealth remains constant over time once the economy opens to international trade, because goods and factor-prices are constant. Since under autarky the economy would still observe a transition to the steady state, 3.3 follows from 3.2 and Proposition 2. Therefore the conclusion is that inside the cone of diversification the "transition" effect is null and there is only the the "international trade" effect of Lemma 1.

Far from the steady state under autarky the "international trade" effect discussed above runs counter to the "transition" effect described in Proposition 2. In particular, even though capital may be the relatively scarce factor in the late-bloomer economy, agents that own an amount of capital larger than the per capita capital will observe an increase in their share of lifetime wealth over the transition to the new steady state with trade. Can the "transition effect" reverse the "international trade" effect? Although a continuity type of argument can be invoked to show that the conclusion of Proposition

[16] To see this, notice that $w_t/r_t = ((1-\eta)/\eta)((1-\nu^*)/(1-l^*))k_t$ for a given level of capital under autarky and the same ratio under trade with specialization in the production of y is $\hat{w}_t/\hat{r}_t = ((1-\eta)/\eta)k_t$. The definitions of ν^* and l^* given before imply that $\hat{w}_t/\hat{r}_t > w_t/r_t$. A similar argument applies for $k_t > \hat{k}_2$.

3 holds for initial capitals close to the cone of diversification, I have been unable to obtain analytical results for arbitrary initial conditions.[17] In stead, I extended the previous analysis using simulations (the numerical method I used is described in the Appendix at the end of the paper). I have simulated the late-bloomer economy under several initial conditions for capital, personal distribution of wealth and parameter values characterizing technologies and minimum consumption requirements. Figures 1 and 2 display the typical evolution of inequality over the transition to the steady state for a late-bloomer economy under autarky and once the economy opens to trade.[18] In all cases I find that the "international trade" effect dominates the "transition" effect: when the economy opens to international trade inequality falls (jumps), and over the transition it remains at a lower (higher) level than what it would under autarky.

Figures 3 and 4 display the evolution of inequality under autarky and under trade for the small economy opening in different dates. That is, the figures offer a picture of what could be gained (or lost) in terms of inequality if the economy opens in period $t+j$ instead of in period t. Figure 3 corresponds to a transition from below. It shows that the latter occurs the opening to international trade, the smaller will be the reduction in inequality and the larger it will be in the long run. Figure 4 corresponds to a transition from above, and it reveals that the latter occurs the opening, the larger will be both the increase and the level of inequality at the steady state. To gain some intuition to understand these findings, it will be convenient to use the Coefficient of Variation (standard deviation over the mean) as the index to measure inequality. It follows from Equation (10) that inequality in the period the economy opens to international trade is given by $cv(\omega_t^i) = (R_t/\omega_t)sd(k_t^i)$, where R_t and ω_t represent the interest factor and average wealth under trade, and where $sd(k_t^i)$ is the standard deviation of capital holdings under autarky. Thus, inequality at the moment of the opening is a function of R_t/ω_t. This ratio is decreasing when the economy converges to the steady state with trade from below, and it is increasing when the economy converges from above. This follows from the fact that the interest factor an average wealth are, respectively, decreasing and increasing in the stock of capital for the given prices of goods in the international markets. Furthermore, Proposition 3 asserts that there will be a fall (jump) in inequality even if the stock of capital is close to k^*. Thus, the falls (jumps)

[17] The main difficulty is that I need to compare equilibrium paths and steady states under autarky and trade. The problem is that the speed of transition is faster the further away the economy is from the steady state (i.e., suggesting that it is faster under autarky than with trade), but at the same time, the speed should be smaller the smaller is the capital share (i.e., suggesting it is faster with trade than under autarky, (see King and Rebelo, 1993)). Thus, I could not exploit the transitivity property of Lorenz orderings as in theorems 2 and 3 in Chatterjee (1994).

[18] For simplicity, the index of inequality I used in the simulations is the Coefficient of Variation in lifetime wealth (standard deviation over the mean). This index to measure the differences in inequality in two distributions is equivalent to Lorenz dominance whenever the two distributions do not cross.

observed in the previous pictures at the moment of the opening reflect the changes over time in R_t/ω_t. These results offer a different picture of the previous finding that the "international trade" effect dominates the "transition" effect: the sooner an economy opens to international trade, the smaller will be the level of inequality in the long run.

5 Financial intermediation and trade

In this section I study the relationship between differences in the financial intermediation and international trade. In particular, I study how differences in the financial intermediation sector may determine the pattern of trade and the evolution of inequality, and how trade may affect the role of financial intermediation.

5.1 The pattern of trade and inequality

For simplicity, I start looking at the effects of differences in the depreciation rate of capital. Latter I study the effects of costly financial intermediation in a more explicit way. Assume that all but one small economy have the same depreciation rate δ as before, and that in the remaining economy $\bar{\delta} > \delta$. Assume also that the opening to trade occurs after all economies in the world have reached the steady state under autarky. I will continue assuming that the economy with larger depreciation rate is small enough to be unable to affect equilibrium prices in the rest of the world, which as before I will treat as a single economy.[19] Since under autarky all economies produce both goods, then all economies devote the same amounts of labor $(l^*, 1-l^*)$ and the same fractions of capital $(\nu^*, 1-\nu^*)$ to the production of intermediate goods x and y. Furthermore, since both economies are at the steady state, for both economies the net return to capital is equal to the rate of time preference $(1-\beta)/\beta$. It follows that $k^* > k^*_{\bar{\delta}}$ because the gross return to capital r is decreasing in capital. Therefore at the steady state under autarky the differences in transaction costs give rise to differences in the stationary stock of capital. If $B = ((1-\nu^*)/(1-l^*))^\eta (l^*/\nu^*)^\theta ((1-l^*)/l^*)$, it follows from Equation (4) that

$$\left(\frac{p_x^*}{p_y^*}\right)_{\bar{\delta}} = \frac{\gamma}{1-\gamma} B (k^*_{\bar{\delta}})^{\eta-\theta} > \frac{\gamma}{1-\gamma} B (k^*)^{\eta-\theta} = \frac{p_x^*}{p_y^*}, \tag{23}$$

thus the differences in the depreciation rate are able to create comparative advantages which will be potentially exploited with international trade. Specifically, for the small economy with larger depreciation rate, in principle it will be profitable to reallocate

[19] The variables corresponding to the economy with a larger depreciation rate will be denoted with a subindex $\bar{\delta}$.

primary factors to increase the production of the labor intensive good y and reduce the production of the capital intensive good x. The interesting question is, how large will be the reallocation of primary factors? Is it possible for the economy with larger δ to reallocate capital and labor and still produce both goods at a steady state with trade? The answer is

Proposition 4. When the economies differ in their depreciation rates the cone of diversification for capital completely collapses.

The proof of Proposition 4 is based on the factor-price equalization theorem (see for instance Samuelson 1996). If prices of goods are the same in both economies then positive production of both goods in both economies and identical technologies for production (absent factor intensity reversals) lead to equal factor prices in both economies, in particular, $r^*_{\bar{\delta}} = r^*$. But since $\bar{\delta} > \delta$, then $r^* + 1 - \bar{\delta} < 1/\beta = r^* + 1 - \delta$, therefore the economy with larger depreciation rate cannot be at a steady state.

The implication of Proposition 4 is that the economy with larger depreciation rate will specialize production in one intermediate good in any equilibrium with international trade. The dynamics under trade for the economy with larger depreciation rate of capital are slightly different from the ones in the previous section. Let $k_{\bar{\delta}y}$ denote the new steady state stock of capital under trade, and suppose that $k_{\bar{\delta}y}$ is larger or equal than the lower end of the diversification cone we found before, k_y. At that level of capital we found that at the equilibrium prices given in international markets $r_y(k_y)+1-\delta = 1/\beta$, so that the economy was at a steady state. Since r_y is decreasing in the level of capital (for the given p^*_y) and $\bar{\delta} > \delta$, it follows that $r_y(k_{\bar{\delta}y}) + 1 - \bar{\delta} < 1/\beta$, that is, $k_{\bar{\delta}y}$ would be too large to be sustainable as a steady state level of capital. The implication is that the new steady state level of capital $k_{\bar{\delta}y} < k_y$. Thus, for $k_{\bar{\delta}y} \leq k \leq k^*$ consumption initially increases after the opening to trade, but over the equilibrium path towards the new steady state both consumption and capital will keep decreasing for all agents.

What are the dynamics for the economy with larger depreciation rate with other initial conditions? If k_0 is sufficiently small the economy would specialize production in the labor intensive good as before, and it would converge to a lower steady state (in terms of capital) than under autarky. To be possible for the economy with a larger depreciation rate to specialize production in the capital intensive good x, its initial capital has to be sufficiently large, but in addition, the corresponding steady state level of capital $k_{\bar{\delta}x}$ must be larger than k^* (otherwise the economy would have comparative advantage in the labor intensive good y). In particular, the following restriction on parameter values must be satisfied:[20]

[20] In (24), $p^*_x = (k^*)^{(\eta-\theta)(1-\gamma)}\gamma(\nu^{*\theta} l^{*1-\theta})^{\gamma-1}((1-\nu^*)^{\eta}(1-l^*)^{1-\eta})^{1-\gamma}$.

$$\bar{\delta} - \delta < \frac{p_x^* \theta}{(k^*)^{1-\theta}} \left(1 - \left(\frac{l^*}{\nu^*} \right)^{1-\theta} \right). \quad (24)$$

When the restriction in (24) is satisfied the steady state with complete specialization in x lies in (k^*, k_x). This means that there are levels of capital such that in the beginning, the small economy may completely specialize production in the capital intensive good. However, over the transition to the steady state the pattern of production will suddenly change, and the economy will completely specialize in the production of the labor intensive good. Of course, for sufficiently different δ and $\bar{\delta}$ the previous condition will be violated and then the only possible steady state involves complete specialization in the labor intensive good.[21] Summarizing:

Proposition 5. With international trade in intermediate goods and $\bar{\delta} > \delta$,
5.1 If $k_0 \geq k_x$ for the economy with a larger depreciation rate and the restriction in (24) is satisfied, then the economy completely specializes production in the capital intensive good and converges from above to $k_{\bar{\delta}x} \in (k^*, k_x)$.
5.2 If $k_0 \geq k_x$ and condition (24) is not satisfied, or if $k^* < k_0 < k_{\bar{\delta}x}$, in the beginning the economy specializes completely in the production of the capital intensive good, over the transition reverses de pattern of specialization, and it converges from above to a steady state $k_{\bar{\delta}y} < k^*$ with complete specialization in the labor intensive good.
5.3 If $k_0 < k^*$ then the economy completely specializes production in the labor intensive good and converges from above to a $k_{\bar{\delta}y} < k^*$ when $k_{\bar{\delta}y} < k_0$, and from below when $k_0 < k_{\bar{\delta}y}$.

With this background in place, I study the effects of differences in the financial intermediation sector. To keep the model tractable I assume that the depreciation rate of capital δ is the same in all economies and that the only difference among them is in the cost to convert units of the investment good into units of new capital. Specifically, I assume that the law of motion for capital takes the form

$$k_{t+1}^i = g(i_t^i) + (1 - \delta) k_t^i, \quad (25)$$

where $g(i_t^i) = (1 - \tau) i_t^i$, with $\tau \in [0, 1)$.[22] Using this specification the parameter τ can be seen as a measure of the transaction cost of financial intermediation.[23] I will

[21] When the economy with a larger depreciation rate opens to trade from the steady state under autarky relative factor prices w/r decrease over the transition to the new steady state. Thus in the sense discussed before the Stolper-Samuelson theorem still holds under the presence of different depreciation rates.

[22] The linearity of g preserves the perfect aggregation property of consumers problems exploited in Section 3. With identical agents one could assume a more general non linear, increasing function g of i_t as for instance in Marcet and Marimon (1992) and the results I derive next would still hold.

[23] Larger transaction costs of intermediation are known to be the result of imperfect information, lack of enforcement of contracts, idiosyncratic risks, and inefficient institutions, among other frictions.

continue assuming that in all but one economy the financial intermediation cost is zero and that for the remaining economy $\tau > 0$.

I show in what follows that under the maintained assumptions Propositions 4 and 5 hold with minor modifications. I start looking at the steady state under autarky for the economy with larger transaction costs. From the first order condition (9) of the consumer's problem at the steady state it follows that: $\beta((1-\tau)r_\tau + (1-\delta)) = 1$, therefore $r^*_\tau = r^*/(1-\tau)$ and $k^*_\tau < k^*$. Once international trade is allowed the factor price equalization theorem applies again and the argument in Proposition 4 rules out the existence of a steady state with positive production of both intermediate goods. Thus for $k_0 \leq k^*$ the economy with larger transaction costs specializes production in the labor intensive good. For this economy to attain the steady state with complete specialization in the capital intensive good, the initial capital must be sufficiently large, and $\tau < 1 - (l^*/\nu^*)^{1-\theta}$ must hold, otherwise the only possible steady state involves specialization in the labor intensive good for all initial conditions.

The results in the last section together with Proposition 5 suggest an interesting effect of international trade on the evolution of wealth inequality. Proposition 5 provides conditions, in addition to relative abundance of factors, under which an economy may initiate a new transition to the steady state from above after the opening to international trade. Suppose that the stock of capital for the small economy at the moment of opening to international trade is such that $k_{\bar{\delta}y} < k_t < k^*_{\bar{\delta}}$. Up to that moment, inequality in the economy has been increasing, but because of the opening to trade with the rest of the world, inequality initially will jump, and then it will decline over the transition to the new steady state. Thus, trade may reverse the dynamics of capital accumulation and wealth inequality when transaction costs are not the same among economies. Notice also that, unlike the original Heckscher-Ohlin model where dynamics of prices and aggregate variables are unambiguously linked to relative factor endowments, the connection between wealth dynamics and factor endowments is less clear once transaction costs in the financial sector are taken into account. For instance, we could think of two small open-economies with similar factor endowments and differing only in their depreciation rates, say $\bar{\delta_1} > \bar{\delta_2}$. If the capital labor ratio in the economies is smaller than in the rest of the world when they open to trade, then the first economy would converge to $k_{\bar{\delta_1}y}$, and the second economy to $k_{\bar{\delta_2}y}$, where $k_{\bar{\delta_1}y} < k_{\bar{\delta_1}y}$. An interesting situation appears when the initial capital labor ratio of the two economies lies between the two steady state levels. In this case inequality would jump and keep declining over the transition in the first economy, whereas it would fall and keep increasing in the second.

5.2 Does trade increase the real income of an economy?

Even if Proposition 5 shows that international trade may be bad in terms of the long run stock of capital in a small open-economy, it is not obvious that the real income of the economy will also shrink in the long run when there are differences in transaction costs. Will trade promote a decline in the real income of such an economy?[24]

To answer this question I assume that the differences between economies are due to a larger depreciation rate (the argument for different transaction costs is analogous) and I compare the present value of the economy at the autarky steady state to its present value under trade, denoted respectively $v(\bar{\delta})$ and $\hat{v}(\bar{\delta})$. To this end, let $\psi(\bar{\delta}) = v(\bar{\delta})/\hat{v}(\bar{\delta})$. Using the technology to produce good z, the equilibrium conditions for l^*, ν^*, and the corresponding expressions in (23), some algebra reveals that

$$\psi(\bar{\delta}) = \frac{y^*}{\hat{y}^*}\frac{1}{1-\gamma}\frac{(p_y^*/p_x^*)_{\bar{\delta}}^{\gamma}}{(p_y^*/p_x^*)^{\gamma}}, \tag{26}$$

where y^*, $(p_y^*/p_x^*)_{\bar{\delta}}$ and $\hat{y}^*$, (p_y^*/p_x^*) stand for domestic production of good y and equilibrium prices under autarky and under trade respectively. The next proposition states conditions under which the value of the economy at the steady state under autarky is larger than that with international trade.

Proposition 6. For all $\bar{\delta}$ sufficiently close to δ, $\psi(\bar{\delta}) > 1$.

Proof: Since $\psi(\bar{\delta})$ is a continuous function of $\bar{\delta}$ it is sufficient to show that $\psi(\delta) > 1$. Using the fact that $\hat{y}^* = k_y^{\eta} = (k^*(1-\nu^*)/(1-l^*))^{\eta}$ when $\bar{\delta} = \delta$ and the economy specializes production in the labor intensive good y, then with the equalities in (23) it follows from (26) that $\psi(\delta) = (1-l^*)/(1-\gamma) > 1$, where the last inequality comes from the definition of l^* and the fact that $\theta > \eta$.

The intuition behind Proposition 6 is that there are costs and benefits associated to the opening to trade for an economy with a larger depreciation rate than in the rest of the world. The costs can be measured by the reduction in the stock of capital due to complete specialization of production in the labor intensive good. The benefits come from the change in relative prices, which increases real income. Therefore the closer is $\bar{\delta}$ to δ, the larger will be the reduction in the steady state level of capital under trade, and at the same time, the smaller will be the change in relative prices. Stated differently, when the difference in relative goods' prices between autarky and

[24] This question is a version of the "immiserizing growth" problem studied first by J. Bhagwati. Bhagwati (1958) showed that the growth process of an economy could deteriorate its terms of trade sufficiently so that the economy would end up with a smaller real income than before the expansion. Here I ask whether the contraction of the economy as a consequence to trade has a negative effect on its real income, even though the contraction does not change the terms of trade.

trade is small, the amount of capital embedded in imports is too small to compensate the actual reduction in capital in the small economy. Thus, the implication is that economies where the cost of financial intermediation is relatively close to that in the rest of the world may actually loose real income. However, Proposition 6 does not need to hold when the differences in the costs of financial intermediation are sufficiently large. A simple numerical example will suffice to make this point. For instance, with $\beta = .99$, $\theta = .8$, $\eta = .4$, $\gamma = .6$, $\delta = .025$ and $\bar{\delta} = .0805$, real income is essentially the same under both, autarky and international trade. The same configuration of parameters but with $\bar{\delta} = .1$ produces $\psi = .8035$, hence real income is larger at the steady state with international trade, even though the stock of capital is smaller than under autarky. This example is interesting because it suggests that international trade is able to alleviate the effects of frictions in the financial intermediation sector.

6 Conclusion

In this paper I develop a dynamic Heckscher-Ohlin model to study international trade effects on personal income inequality and the interaction between international trade and the financial intermediation sector. In my model, transitions to steady states from below (from above) promote an increase (a decline) of inequality in the personal distribution of wealth. International trade, on the other hand, promotes a decline (an increase) in the functional distribution of income if the economy converges to the steady state from below (from above). My results suggest that the effect of international trade on the personal distribution of wealth dominates the effect of the transition.

I also show that trade favors complete specialization of production in labor intensive goods in economies where financial intermediation is more costly. Thus, the financial intermediation sector may have a key role on the determination of the pattern of production in a small open economy. Furthermore, I find that economies with larger costs of financial intermediation are likely to benefit more from specialization and trade than economies where these costs are only slightly larger than in the rest of the world. In fact, these latter economies converge to a steady state where the present value of their production is smaller than under autarky.

The results in this paper show that the evolution of wealth inequality may not always increase or decline following the opening to international trade. It would be interesting to incorporate exogenous growth into the model. In such a model wealth inequality would increase in economies with higher than average growth rates, and it would decrease in economies with growth rates smaller than average. Testing such a model in the data would shed light on the relationship between growth and inequality, and indirectly, between international trade and inequality. This extension is left for future work.

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7 Appendix

Proof of Lemma 1: For a given stock of capital k in operation at period t the ratio of equilibrium interest rates under autarky (r) and under trade ($\hat{r}$) when the economy specializes production in the labor intensive good y satisfies:

$$\frac{r}{\hat{r}} = \frac{p_y}{p_y^*}\left(\frac{1-\nu^*}{1-l^*}\right)^{\eta-1},$$

where p_y and p_y^* stand for equilibrium prices of good y under autarky and trade respectively (time subscripts are dropped to simplify notation). $p_y/p_y^* = (k/k^*)^{(\theta-\eta)\gamma}$, thus p_y/p_y^* is increasing in k and $r/\hat{r} = 1$ if $k = \hat{k}_1$. The definitions of ν^* and l^* imply that $\hat{k}_1 < k_y$. It follows that $r/\hat{r} > 1$ for $\hat{k}_1 \leq k \leq k_y$. For $k_y \leq k \leq k^*$ the same conclusion holds because inside the cone of diversification $\hat{r} = (1-\beta(1-\delta))/\beta$, i.e. the equilibrium interest rate at the steady state under autarky. Similar arguments apply to show that $w/\hat{w} < 1$ for all $k \leq k^*$. With the definition of $\hat{k}_2$ the same reasoning can be used to establish the results corresponding the case of $k^* \leq k \leq k_x$ where the economy specializes production in the capital intensive good x.

Proof of Proposition 3: From Lemma 1 $\hat{k}_1 < k_y$ and that $\hat{k}_2 > k_x$, thus for $k_y \leq k_0 < k^*$ ($k^* < k_0 \leq k_x$) Lemma 1 applies. Furthermore, since k_0 is inside the cone of diversification $\hat{w}$ jumps and $\hat{r}$ falls to the corresponding stationary levels under trade. Next, let $\omega_0^i/\omega_0 = Ns_0^i$ denote the ratio of agent i wealth to average wealth immediately before the economy opens to trade and $\hat{\omega}_0^i/\hat{\omega}_0 = N\hat{s}_0^i$ under the new equilibrium prices with trade and assume that $k_y \leq k_0 < k^*$. Choose an agent with $k_0^i < k_0$ and assume that for that agent $s_0^i \geq \hat{s}_0^i$. It follows that

$$k_0^i\left[\sum_{j=0}^{\infty}\frac{\hat{w}_j}{\prod_{s=0}^{j}\hat{R}_s} - \sum_{j=0}^{\infty}\frac{w_j}{\prod_{s=0}^{j}R_s}\right] \geq k_0\left[\sum_{j=0}^{\infty}\frac{\hat{w}_j}{\prod_{s=0}^{j}\hat{R}_s} - \sum_{j=0}^{\infty}\frac{w_j}{\prod_{s=0}^{j}R_s}\right]. \qquad (27)$$

The term in square brackets is strictly positive when $k_y \leq k_0 < k^*$, since over the equilibrium path under autarky $w_t \leq \hat{w}$ and $R_t \geq \hat{R}$ $\forall t$. Thus (27) contradicts the fact that $k_0^i < k_0$. Therefore $s_0^i < \hat{s}_0^i$. A similar argument applies to show that $s_0^i > \hat{s}_0^i$ if $k_0^i > k_0$. Also, by construction $s_0^i = \hat{s}_0^i$ if $k_0^i = k_0$. Next, order agents according to increasing wealth and denote by $\bar{i}$ the agent with the largest wealth share smaller or equal to average wealth in the economy. It is straight forward to check that $s_0^i \leq s_0^{i+1}$ implies that $\hat{s}_0^i \leq \hat{s}_0^{i+1}$. It follows that $\sum_{i=1}^{J} s^i < \sum_{i=1}^{J}\hat{s}^i$ $\forall 1 \leq J \leq \bar{i}$, and that $\sum_{i=1}^{N-J-1} s^i = 1 - \sum_{i=N-J}^{N} s^i \leq 1 - \sum_{i=N-J}^{N}\hat{s}^i = \sum_{i=1}^{N-J-1}\hat{s}^i$ for all $J \leq N - \bar{i}$. Thus $\sum_{i=1}^{J} s^i \leq \sum_{i=1}^{J}\hat{s}^i$ for all $J \leq N$, with equality only for $J = N$. Therefore if $k_y \leq k_0 < k^*$ inequality falls and remains constant after the opening to trade. Since

$\alpha < 0$, under autarky inequality would continue increasing up until the corresponding level at the steady state. The argument for the case of $k^* < k_0 \leq k_x$ is analogous.

Computation

A brief description of the numerical method is as follows: starting from an arbitrary function v_0 of the state k for the value function, perform iterations on the Bellman Equation associated to the corresponding planner's problem on a grid of points. The decision rule for capital accumulation is approximated with piecewise linear functions between grid points. Thus with this method the corresponding version of the Euler Equation holds exactly at points in the grid (see for instance Huggett (1993) for further details). [25] Once the decision rule for capital has approximately converged I simulate a transition towards the steady state over 1000 periods and I compute the objects of interest. To simplify the computation of inequality over time I use the coefficient of variation (standard deviation over the mean). This measure is convenient because it can be computed recursively as

$$cv(\omega_{t+1}^i) = cv(\omega_t^i)\frac{\beta}{\beta - (a({}_tR)/\omega_t)}.$$

The computation is further simplified because I need to solve only a planner's problem for a given initial distribution of capital, given that $cv(\omega_0^i) = R_0 sd(k_0^i)/\omega_0$, and prices and average wealth can be recovered from the optimal allocation.

The parameter values used in the benchmark simulations are described next. I use $\beta = .99$ and $\delta = .025$ which are commonly used in applied work for the U.S. economy simulating quarterly data. For the parameters in the technologies I use $\gamma = .5$, $\theta = .38$ and $\eta = .34$. These parameter values produce a capital share about .36 which is again standard in the quantitative literature. I have computed transitions with several values for the parameters above, α, and initial initial distributions.

[25] In practice I use 1300 evenly spaced points in the grid. Computing time is small given that there is only one state variable.

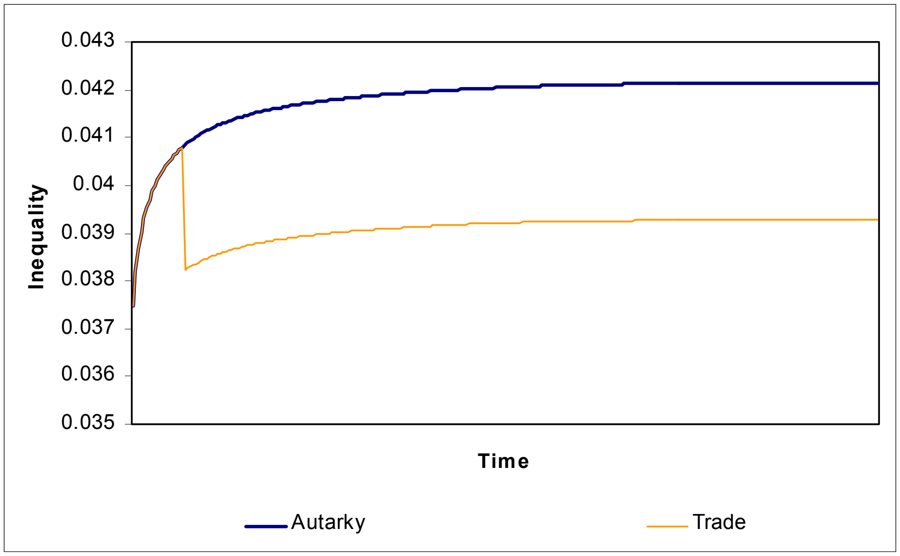

Figure 1: Evolution of inequality under autarky and trade along a transition from below.

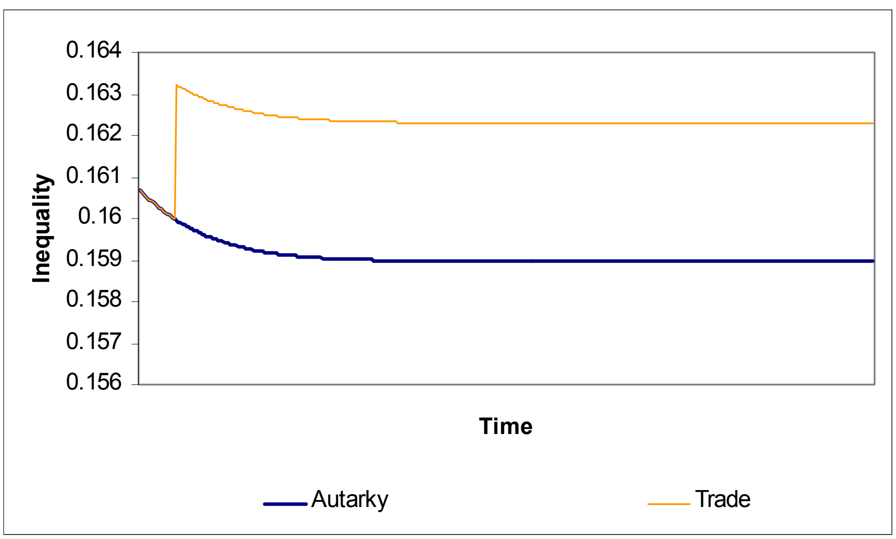

Figure 2: Evolution of inequality under autarky and trade along a transition from above.

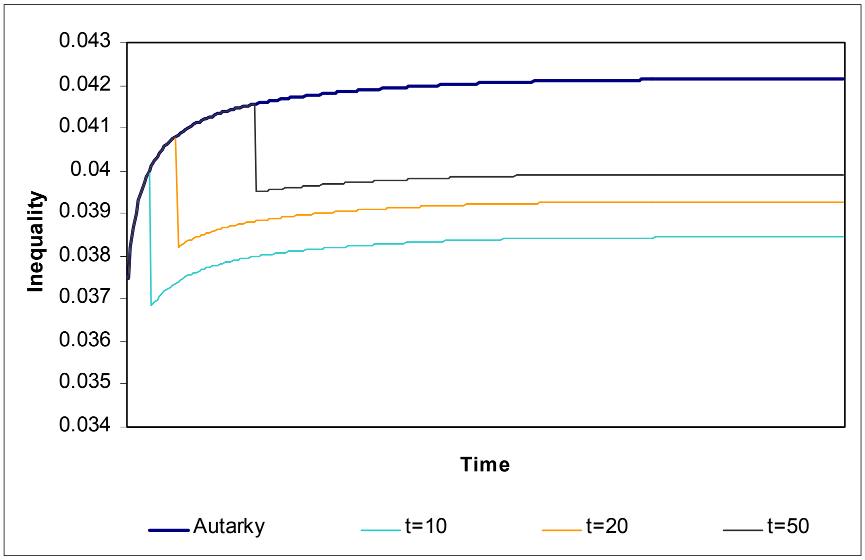

Figure 3: Timing effects of the opening to international trade along a transtition from below.

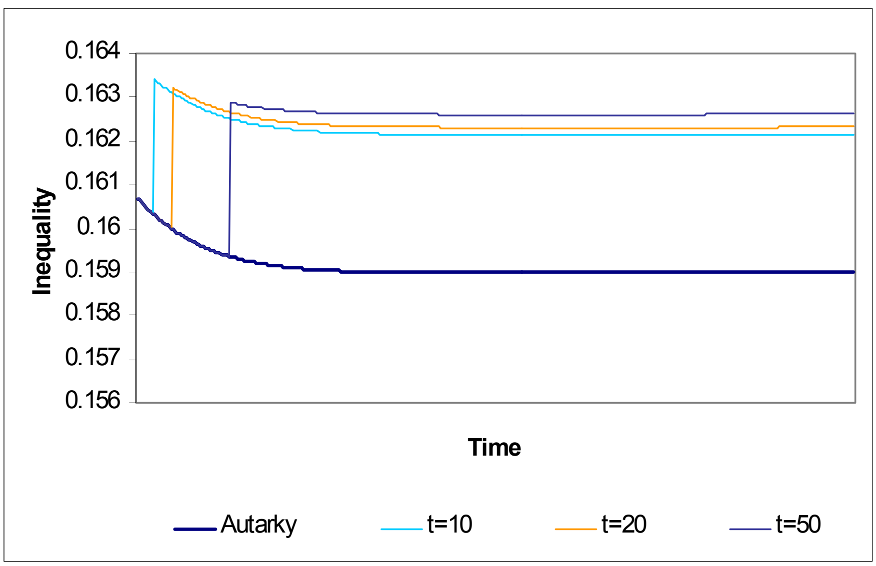

Figure 4: Timing effects of the opening to international trade along a transition from above.